HW #1
Due date: 10/6/08

Problem #1:
A) Consider a cantilever beam loaded by a concentrated load at the tip (Figure 1). By using strength of materials formulas, identify the spring constant \( k = \frac{F}{x} \), the deflection for a mass \( m = \rho b^3 \) and the relative deflection \( \delta_m = \frac{x}{L} \). Using the concepts discussed in class show that \( \delta_m \) is proportional to \([S]\), in which \([S]\) is a characteristic dimension.

B) Consider the same cantilever beam but this time loaded at the tip by a capillary force as illustrated in Figure 2. This force is \( F_c = \gamma \pi R \cdot 2 \), where \( \gamma \) is the surface tension and \( R \) is the radius of the ring in contact with the fluid. Compute the relative deflection \( \delta_c = \frac{x}{L} \), \( x \) obtained from \( F_c \) and the corresponding formula. Show that \( \delta_c \) scales as \([S^{-1}]\).

C) Compute the ratio \( \frac{\delta_m}{\delta_c} = KB_o \) (note: here \( K = \frac{b}{4\pi R} \) rather than the \( k \) in A) and B)) and show that the Bond number \( (B_o) \) is given by \( B_o = \frac{\rho gb^2}{\gamma} \). What is the physical meaning of \( B_o \)?

Problem #2:
This problem deals with the scaling of minimal analytical sample size. One can show that the minimum volume of sample required to detect a given analyte concentration is:

\[
V = \frac{1}{\eta N_A C_i}
\]

where \( \eta \) is the sensor efficiency between 0 and 1, \( N_A \) is Avogadro’s number (6.02x10^{23} \text{ mol}^{-1}) and \( C_i \) is the analyte concentration (moles/L). After reading Petersen et al., 1998, and Manz et al., 1990, explain how the graphs of target concentration analyte vs. required sample size reported in these papers were obtained. What is the minimum sample size for accurate detection of DNA? Is this result expected or counterintuitive?


Problem #3:
This problem deals with flow at low Reynolds numbers. Read Chapter 6 of “Life's Devices” by Steve Vogel and then write a paragraph or two summarizing the physical mechanisms used by microorganisms and blood cells to move in liquid and/or capillaries.

Problem #4: Read chapter 2 of the book by Senturia.

A Library Treasure Hunt: By consulting recent issues of at least two of the following MEMS-oriented journals, locate articles that illustrate (a) mechanical sensing, (b) use of MEMS for optics, (c) a chemical or biological system, and (d) an actuator. The journals are: IEEE/ASME journal of Microelectromechanical Systems, Journal of Micromechanics and Microengineering,

**Problem #5:**
A Library Treasure Hunt: Consult the references given thus far (under “Related Reading” in Chapters 1 and 2, and in Problem 1.1 in “Microsystem Design”) and find an example of a microsystem or MEMS device that has been modeled using block-diagram representations, and an example of a microsystem that has been modeled with equivalent-circuit lumped-element models. Explain how the elements of the model correspond to the elements of the device in question. (This is Problem 2.1 in “Microsystem Design”).

**Note:** Make sure to provide your own individual answers to all problems.

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**Figure 1**

**Figure 2**

Water